Nuclear Physics

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1 Basic properties of the nucleus

1.1 Fermions and bosons

Elementary particles are classified as either **fermions** or **bosons**. Fermions satisfy the Pauli exclusion principle, which states that no two fermions have the same wave function, they obey the Fermi-Dirac statistics of statistical mechanics. For a fermion s takes one of the values $\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots$.
Bosons are particles which obey Bose-Einstein statistics, and are characterised by the property that any number of particles may be assigned the same single-particle wave-function. For a boson s takes one of the values $0, 1, 2, \ldots$.

1.2 The particle physicist's picture of nature

Four types of interaction field may be distinguished in nature:

	Relative strength	Range
Strong field	1	Short, $\sim$1 fm$= 10^{-15}$ m
Electromagnetic field	$\sim 10^{-2}$	Long, $\propto \frac{1}{r^2}$
Weak field	$\sim 10^{-14}$	Short, $\sim 10^-2$ fm$= 10^{-17}$ m
Gravitational field	$\sim 10^{-44}$	Long, $\propto \frac{1}{r^2}$

The particles associated with the interaction fields are bosons.

Force	Carried by	Acts on
Strong field	Gluon	Quarks and Gluons
Electromagnetic field	Photon	Quarks, Charged leptons, W^+, W^-
Weak field	W^+, W^-, Z^0	Quarks and Leptons
Gravitational field	Graviton (not observed yet)	All

1.3 Units

Every branch of physics tends to find certain units particularly congenial.

Length	Fermi (or femtometre)	1 fm$= 10^{-15}$ m
Energy	Electron-volt	1 eV$= 1.6 \times 10^{-19}$ J
Nuclear cross-section	Barn	1 b$= 100$ fm$^2 = 10^{-28}$ m^2
Mass	Unified atomic mass unit	1 u$= 1.66 \times 10^{-27}$ kg
	Weird energy unit	1 u$= 931.494$ MeV/c^2

1.4 Properties of the proton, neutron and the electron

	Charge [C]	Mass [u]	E_0 [MeV]	Spin [$\hbar$]	Magnetic moment [J/T]
Proton	$e = 1.6022 \times 10^{-19}$	1.007276	938.3	$\frac{1}{2}$	1.411×10^{-26}
Electron	$-e = -1.6022 \times 10^{-19}$	0.000549	0.511	$\frac{1}{2}$	9.28×10^{-24}
Neutron	0	1.00866	939.6	$\frac{1}{2}$	-9.66×10^{-27}

1.5 The quark model of nucleons

Nowadays we divide the elementary fermions into two families of 6 members:

Quarks	up, u	charm, c	top, t	$\frac{2}{3}e$
	down, d	strange, s	bottom, b	$-\frac{1}{3}e$
Leptons	electron, e^-	muon, μ^-	tau, τ^-	e
	electron neutrino, ν_e	muon neutrino, ν_μ	tau neutrino, ν_τ	0

A proton consists of two up quarks and one down quark (uud) and the neutron consists of two down quarks and one up quark (ddu).

2 The nuclear chart

2.1 The nucleus

The notation of an atom is as follows:

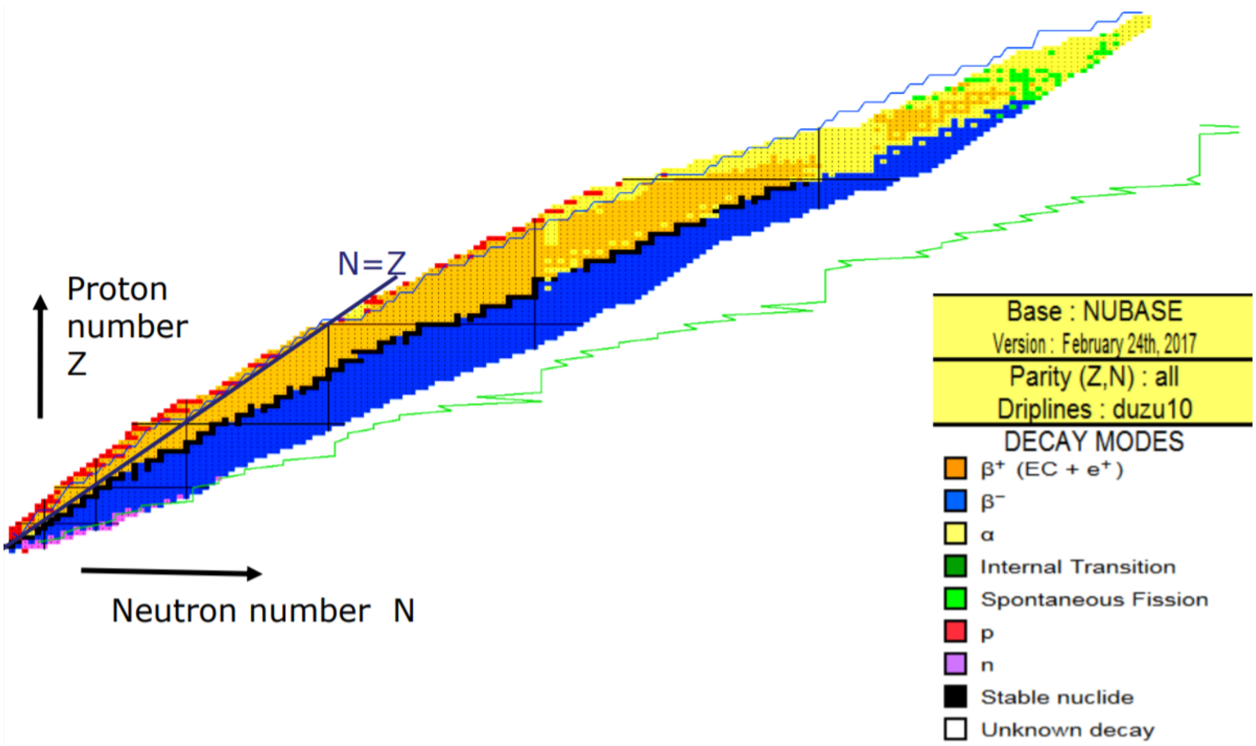
$$^{A}_{Z}\mathrm{El}_{N}$$

The nucleus is formed by A nucleons, this is the **mass number**. This number corresponds to the sum of the number of protons and the number of neutrons:

$$A = Z + N$$

Here Z is the number of protons, or the **atomic number**, and N is the number of neutrons. A neutral atom has the same number of protons and electrons, thus the number of electrons is also given by Z.

2.2 The nuclear chart



2.3 Isotopes, isotones, isobars, isomers and isodiasphers

Isotopes: nuclei with the same number of protons but a different number of neutrons
Z and El stay the same, A and N change.

Isotones: nuclei with the same number of neutrons
N stays the same, Z, A and El change.

Isobar: nuclei with the same mass number, but different numbers of protons and neutrons
A stays the same, Z, N and El change.

Isodiaspher: nuclei with different numbers of protons and neutrons and a different mass number, but the same difference of neutrons and protons (neutron excess)
$N - Z$ stays the same, A, Z, N and El change.

Isomer: nuclei with the same number of protons and neutrons (same isotope), but different energy state A, Z, N and El stay the same, the energy state changes.

2.4 Masses and binding energies

The mass of an atom can be calculated by adding up the masses of its protons, neutrons and electrons minus the binding energy of the atom:

$$M_{\text{atom}} = N \cdot m_{\text{neutron}} + Z \cdot m_{\text{proton}} + Z \cdot m_{\text{electron}} - \frac{(B_{\text{atom}} + B_{\text{nucleus}})}{c^2}$$

The **mass excess** of a nuclide is the difference between its actual mass and its mass number in atomic mass units (comparism with C-12).

$$\Delta M(Z, A) = M_{\text{atom}}(Z, A) - A$$

The **mass defect** is the difference between the predicted mass and the actual mass of an atom's nucleus. The binding energy of a system can appear as extra mass, which accounts for this difference.

$$\Delta M = \frac{(B_{\text{atom}} + B_{\text{nucleus}})}{c^2} = N \cdot m_{\text{neutron}} + Z \cdot m_{\text{proton}} + Z \cdot m_{\text{electron}} - M_{\text{atom}}$$

2.5 The liquid drop model

Or the Bethe-Weizsäcker formula or semi-empirical mass formula.
The Bethe-Weizsäcker formula gives a good approximation for atomic masses and thereby other effects.

$$B(N, Z) = aA - bA^{\frac{2}{3}} - \frac{dZ^2}{A^{\frac{1}{3}}} - s\frac{(N - Z)^2}{A} - \frac{\delta}{A^{\frac{1}{2}}}$$

with $a = 15.835\text{MeV}, b = 18.33\text{MeV}, s = 23.20\text{MeV}, d = 0.7140\text{MeV}$ and

$$\delta = \begin{array}{l} + 11.2\text{MeV for odd } - \text{ odd nuclei (odd N odd Z)} \\ 0 \text{ for even } - \text{ odd nuclei (even N odd Z, or even Z odd N)} \\ - 11.2\text{MeV for even } - \text{ even nuclei (even N and even Z)} \end{array}$$

The terms of the Bethe-Weizsäcker formula represent the following:

1. aA is the **volume term**. It is the major contribution resulting from the attractive strong nuclear interactions.
2. $bA^{\frac{2}{3}}$ is the **surface term**. It represents the loss of binding energy of the nucleons near the surface.
3. $\frac{dZ^2}{A^{\frac{1}{3}}}$ is the **coulomb term**. The electrostatic repulsion of the protons decreases the binding energy
4. $s\frac{(N-Z)^2}{A}$ is the **asymmetry (isospin) term**. Light nuclei have approximately equal proton and neutron numbers. For a heavy nucleus, on the other hand, an excess of neutrons is needed to bring more binding energy. The excess of neutrons decreases the total binding energy.
5. $\frac{\delta}{A^{\frac{1}{2}}}$ is the **pairing term**. Protons and neutrons form pairs with anti-parallel spins. This pairing correlation increases the binding energy

The first 3 terms correspond to macroscopic classic effects arising directly from the liquid-drop model. The other 2 terms are of quantum origin.

2.6 Nuclear shell model

2.6.1 Nuclear potential well

Each neutron moves independently in common potential well that is spherical average of the nuclear potential produced by all other nucleons.
Each proton moves independently in common potential well that is spherical average of the nuclear potential produced by all other nucleons plus the Coulomb potential of the other protons. The coulomb potential is given by:

$$U_c(r) = \begin{cases} \frac{(Z-1)e^2}{4\pi\epsilon_0 R}\left[\frac{3}{2} - \frac{r^2}{2R^2}\right] & \text{for } r < R \\ \frac{(Z-1)e^2}{4\pi\epsilon_0 r} & \text{for } r > R \end{cases}$$

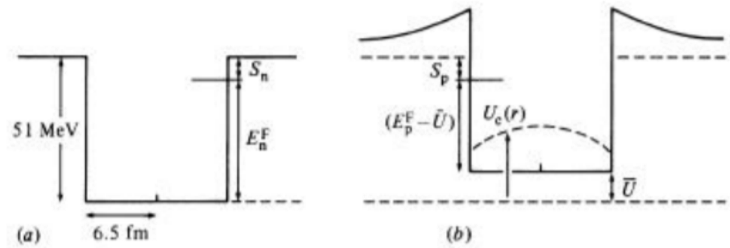


Fig. 5.1 A schematic representation of (a) the neutron potential well and (b) the proton potential well for the nucleus $^{208}_{82}$Pb. E_n^F and $(E_p^F - \bar{U})$ have been estimated using equation (5.5). The observed neutron separation energy S_n of 7.4 MeV implies a neutron well-depth of 51 MeV. The observed proton separation energy S_p of 8.9 MeV implies that $\bar{U} = 11$ MeV. $\bar{U}$ represents the sum of the mean electrostatic potential and the asymmetry energy.

Nuclei with neutron excess: the contribution of the strong nucleon-nucleon interacting to the potential is more attractive for protons than for neutrons, thus the proton well depth relative to $U_c(r)$ is lowered.

2.6.2 Nucleon energies

Pauli principle! Neither two proton nor two neutrons are in the same state.
The Fermi energies E_p^F and E_n^F define the highest occupied energy levels in the ground state.
The integrated density of states is given by

$$\mathcal{N}(E) = \frac{V}{3\pi^2}\left(\frac{2mE}{\hbar^2}\right)^{3/2}$$

Where V is the volume

$$V = 4\pi\frac{R^3}{3}$$

The number of protons and neutrons is given by:

$$N \approx \frac{V}{3\pi^2}\left(\frac{2m_n E_n^F}{\hbar^2}\right)^{3/2} \qquad Z \approx \frac{V}{3\pi^2}\left(\frac{2m_p\left(E_p^F - \overline{U}\right)}{\hbar^2}\right)^{3/2}$$

For nuclei with $A \leq 40$: $N \approx Z$.

$$\rho \approx \frac{N}{V} \approx 0.085\ \text{fm}^{-3} \rightarrow E_n^F \approx 38\ \text{MeV}$$

Neutron separation energy:

$$S_n(N, Z) = B(N, Z) - B(N - 1, Z)$$

Proton separation energy:

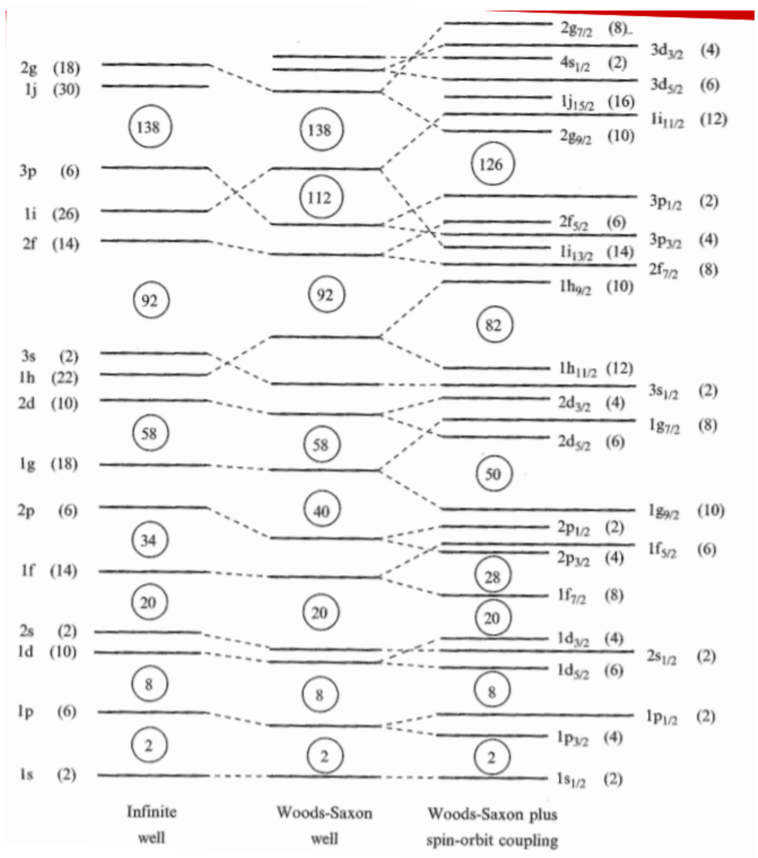
$$S_p(N, Z) = B(N, Z) - B(N, Z - 1)$$

(about 8 MeV).

Velocity of a nucleon at the Fermi energy and the nuclear radius, sets the nuclear time scale:

$$t_{\text{nuc}} = \frac{2R}{v_F} \approx 2.6 \times 10^{-23} \cdot A^{1/3} \text{ s}$$

2.6.3 Nuclear shells



The closed shells are indicated by the "magic numbers" of nucleons;

$$2, 8, 20, 28, 50, 82, 126$$

2.6.4 Notation

There are three labels for orbital labeling: n, L and J.
n indicates the nodes, with $n = 0, 1, 2, \ldots$
L is the angular momentum, with $L = s, p, d, f, g, h, \ldots$
The parity is given by $\pi = (-1)^L$
J is the total angular momentum, with $|L - s| < J < |L + s|$
The multiplicity of the state is $2J + 1$
Spin parity: j^π

2.6.5 Magnetic dipole moment

Total magnetic moment operator for a single nucleon:

$$\boldsymbol{\mu} = \boldsymbol{\mu}_L + \boldsymbol{\mu}_s = \mu_N [g_L \mathbf{L} + g_s \mathbf{s}]/\hbar$$

where $g_L = 1$ for a proton and $g_L = 0$ for a neutron.

$$\boldsymbol{\mu} = \mu_N \left[\frac{1}{2}(g_L + g_s)(\mathbf{L} + \mathbf{s}) + \frac{1}{2}(g_L - g_s)(\mathbf{L} - \mathbf{s}) \right] /\hbar$$
$$\mu = \mu_N \left[\frac{1}{2}(g_L + g_s)j + \frac{1}{2}(g_L - g_s)\frac{(l-s)(l+s+1)}{(j+1)} \right]$$

Since $s = \frac{1}{2}$ and $j = l \pm \frac{1}{2}$, the contribution of the unpaired nucleon is

$$\mu = \begin{cases} \mu & = \mu_N \left[j g_L - \frac{1}{2}(g_L - g_s) \right] \text{ for } j = l + \frac{1}{2} \\ \mu & = \mu_N \left[j g_L - \frac{j}{2(j+1)}(g_L - g_s)\frac{(l-s)(l+s+1)}{(j+1)} \right] \text{ for } j = l - \frac{1}{2} \end{cases}$$

3 Radioactive decay

3.1 Half-life and energy

The decay rate of a nuclear reaction is given by

$$N = N_0 e^{-\lambda t}$$

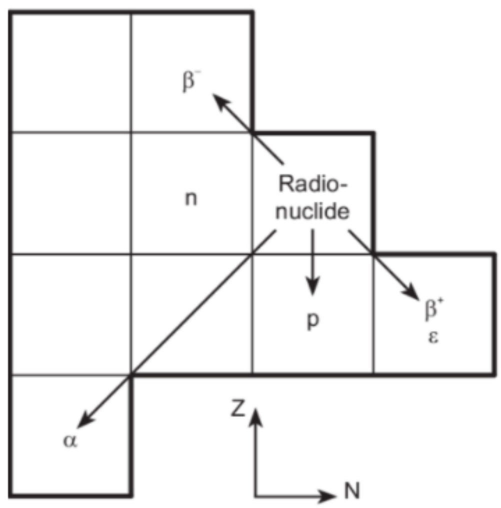
And the half-life of an atom is given by

$$t_{1/2} = \frac{\ln 2}{\lambda}$$

The Q value for a reaction is the amount of energy absorbed or released during the nuclear reaction. It is given by the difference in kinetic energies K of the final (f) and initial state (i), mass difference, and also difference in binding energies B.

$$Q = K_f - K_i = (m_i - m_f)\, c^2 \qquad \text{or} \qquad Q = B_f - B_i$$

3.2 Decays in the nuclear chart



3.3 Alpha decay

With alpha decay a nucleus emits a $^4_2$He nucleus (an α-particle), which reduces the mass number by 4 and the proton number by 2.

$$\text{Example: } {}^{222}_{86}\text{Rn} \rightarrow {}^{218}_{84}\text{Po} + \alpha$$

The generic formula is given by:

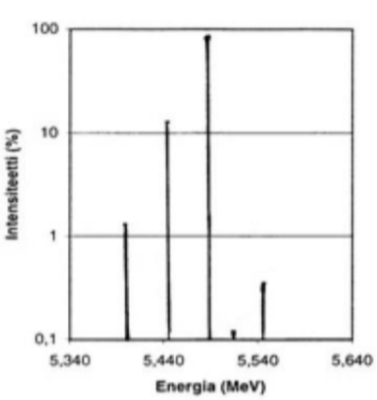
$${}^{A}_{Z}M \rightarrow {}^{A-4}_{Z-2}D + {}^{4}_{2}\text{He}$$

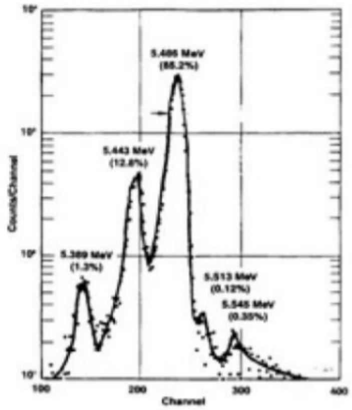
And the Q for this reaction will be

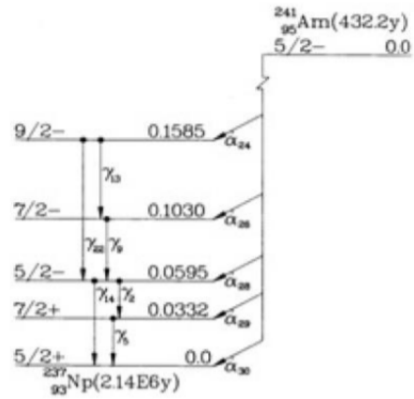
$$Q_\alpha = (m_M - m_\alpha - m_D) \cdot c^2$$

3.3.1 Spectrum

The spectra of alpha decays show discrete energies:

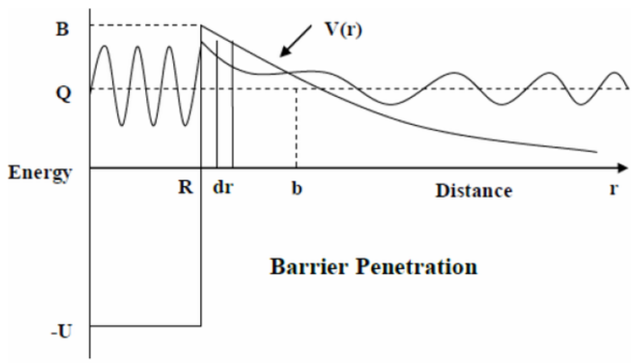






3.3.2 Semi classical description of alpha decay

α-decay can be considered as a quantum mechanical tunneling process.



The α particle has to tunnel through the Coulomb barrier from R to b to escape the mother nucleus. The binding energy gain of $\sim$ 28 MeV when an α particle is formed overcomes the $\sim$ 6 MeV required to remove a nucleon. The height of the barrier is given by the coulomb potential:

$$B = V(C)$$

The distance b is given by:

$$b = \frac{Z_\alpha Z_D e^2}{4\pi\epsilon_0 Q_\alpha}$$

The particle's kinetic energy is:

$$T_\alpha = \frac{M_D}{M_D + M_\alpha} Q_\alpha \approx Q_\alpha$$

which takes into account the recoiling daughter. The larger Q_α,

- the thinner the barrier,
- the higher the penetration probability,
- the shorter the half-life

3.3.3 Tunneling probability

Probability that the α particle tunnels through:

$$p = \exp\left(-2\int_R^b K\,dr\right) = \exp\left(-2G\right)$$

Here G is the Gamow factor:

$$G = \frac{1}{\hbar}\int_R^b \sqrt{2M_\alpha\left(V_{(r)} - T_\alpha\right)}\,dr$$

If $V_{(r)} > T_\alpha$ and $b \gg R$, then:

$$G = \sqrt{\frac{8M_\alpha c^2}{c^2\hbar^2 Q}}\frac{Z_D e^2}{4\pi\epsilon_0}\left(\frac{\pi}{2} - 2\sqrt{\frac{R}{b}}\right)$$

The α-decay rate [s^{-1}] can be calculated as follows:

$$\lambda_\alpha = f \cdot p = f \cdot \exp\left(-2G\right)$$

which relates to the α-decay half-life [s]:

$$T_{1/2} = \frac{\ln 2}{\lambda_\alpha}$$

and to the mean life time [s]:

$$\tau = \frac{1}{\lambda_\alpha} = \frac{1}{fp}$$

3.4 Isobaric disintegrations

Puzzle – broad spectrum of the beta decay, postulation and later discovery of the neutrino/ antineutrino
Cut of the energy surface by a plane of equation N + Z = A (A = constant) shows chains of isobaric disintegrations leading to more stable isobars

3.4.1 Beta - decay

With beta - decay a nucleus emits an electron (a β^- particle) and an antineutrino, $\bar{\nu}_e$, the mass number remains the same, the proton number increases by 1:

$$\text{Example: } {}^{209}_{82}\text{Pb} \rightarrow {}^{209}_{83}\text{Bi} + \beta^- + \bar{\nu}_e$$

The generic formula is given by:

$${}^{A}_{Z}M \rightarrow {}^{A}_{Z+1}D + e^- + \bar{\nu}_e$$

The excess of energy in the final state is shared between 3 bodies: the daughter nucleus , the electron and the anti-neutrino. Because of that, their electron energy spectrum is continuous.

$$\begin{aligned} Q &= \left(M_{\text{nucl,mother}} - M_{\text{nucl,daughter}} - m_e\right)c^2 \\ &= \left(M_{\text{atom,mother}} - M_{\text{atom,daughter}}\right)c^2 \end{aligned}$$

3.4.2 Beta + decay

With beta + decay a nucleus emits a positron (a β^+ particle) and a neutrino, ν_e, the mass number remains the same, the proton number decreases by 1:

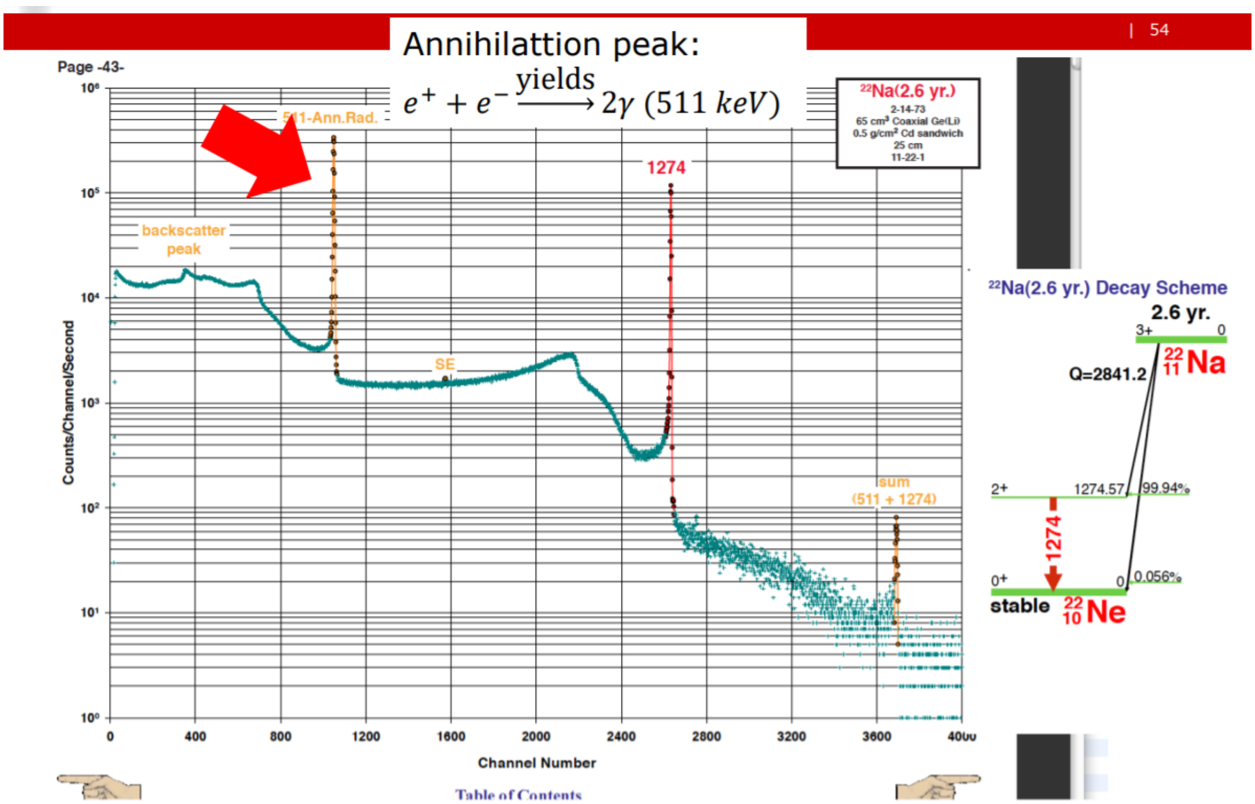
$$\text{Example: } {}^{18}_{9}\text{F} \rightarrow {}^{18}_{8}\text{O} + \beta^+ + \nu_e$$

The generic formula is given by:

$${}^{A}_{Z}M \rightarrow {}^{A}_{Z-1}D + e^+ + \nu_e$$

Results from the transformation, inside the nucleus, of a proton into a neutron, with the emission of a positron and a neutrino (reaction energetically impossible in vacuum):

$$\begin{aligned} Q &= \left(M_{\text{nucl,mother}} - M_{\text{nucl,daughter}} - m_e\right) c^2 \\ &= \left(M_{\text{atom,mother}} - M_{\text{atom,daughter}} - 2m_e\right) c^2 \end{aligned}$$



Corresponding γ-spectrum.

3.4.3 Electron capture

The nucleus captures an electron from one of the inner electron shells, the mass number remains the same, the proton number decreases

$$\text{Example: } {}^{202}_{81}\text{Tl} + e^- \rightarrow {}^{202}_{80}\text{Hg}$$

The generic formula is given by:

$$e^- + {}^{A}_{Z}M \rightarrow {}^{A}_{Z-1}D$$

The energy for this reaction is given by:

$$\begin{aligned} Q &= \left(M_{\text{nucl,mother}} - M_{\text{nucl,daughter}} + m_e\right) c^2 \\ &= \left(M_{\text{atom,mother}} - M_{\text{atom,daughter}}\right) c^2 \end{aligned}$$

3.4.4 Fermi theory of beta decay.

Consider:

$${}^{17}_{9}\text{F} \rightarrow {}^{17}_{8}\text{O} + e^+ + \nu_e$$

Proton in the closed shell cannot change into a neutron since the neutron shell is also full, Pauli principles forbids the transition, thus nucleons from closed shells do not take part in the transition.

- the initial state $\Psi_0 = \psi_p(r_p)$, where ψ_p is the state of the proton in the $d_{5/2}$ shell
- the finial state: $\Psi_f = \psi_n(r_n)\psi_e(r_e)\psi_\nu(r_\nu)$
- the transition rate from Ψ_0 to Ψ_f is $\frac{2\pi}{\hbar}|H_{f0}|^2 n_f(E_0)$

H_{f0} is the matrix element linking the final and initial state, $n_f(E_0)$ is the density of states in Ψ_f at the energy E_0 released in the decay.

Fermi decay. Fermi theory: contact interaction, reference to spin has been suppress as all particles involved are fermions with $s = 1/2$.

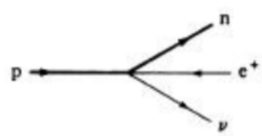


Fig. 12.4 β-decay of a proton in a nucleus as a 'contact' interaction.

Fermi interaction: no change in nucleon spins, allowed transition the positron and neutrino angular momenta must combine to give a total lepton angular momentum of zero.

$$H_{f0}^F = G_w \int \psi_n^\dagger(r)\psi_p(r_p)d^3r \times (\text{lepton part})$$

The nuclear matrix element is

$$M_F = \int\int \psi_n^\dagger(r)\psi_p(r_p)d^3r$$

And G_w is given by terms of constants of particle physics

$$G_w = G_F V_{ud}$$

Fermi constant:

$$G_F = 1.16639 \times 10^{-11}\,(\hbar c)\ \text{MeV}^{-2}$$

Gamow Teller decay. Full theory: interaction is mediated by a W-boson

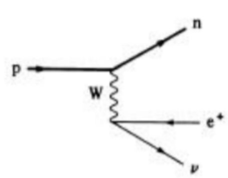


Fig. 12.5 β-decay of a proton in a nucleus mediated by the exchange of a virtual W boson.

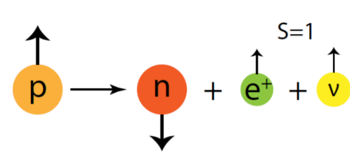


Figure 2.2: Gamow-Teller decay. The spins of the emitted positron and neutrino couple to total spin S = 1, while the angular momentum change of the nucleus is $\Delta J = 0, \pm 1$.

Transition $\Delta j = 0 \bigvee 1$
Total lepton angular momentum J with $j = 1$ and a nuclear part of the interaction summarized in the Pauli operator $\boldsymbol{\sigma}$

$$H_{f0}^{GT} = g_A G_w \int \psi_n^\dagger(r)\boldsymbol{\sigma}\psi_p(r_p)d^3r \times (\text{lepton part})$$

$$\boldsymbol{M}_{GT} = g_A \int \psi_n^\dagger(r)\boldsymbol{\sigma}\psi_p(r_p)d^3r = (M_{GT}^x, M_{GT}^y, M_{GT}^z)$$

g_A is the axial coupling constant.
The mean life is given by:

$$\frac{1}{\tau} = \frac{G_F^2 V_{ud}^2 m_e^5 c^4}{2\pi^3\hbar^7}\left[|M_F|^2 + |M_{GT}^x|^2 + |M_{GT}^y|^2 + |M_{GT}^z|^2\right] f(Z_d, E_0)$$

3.4.5 Selection rules

Spin for e and ν: s= 1/2
- spins parallel: S= 1 "Gamow-Teller decay"
- spins anti-parallel: S=0 "Fermi-decays"

		Fermi		GT	
Type	L	ΔI	Δπ	ΔI	Δπ
Allowed	0	0	No	(0), 1	No
First Forbidden	1	(0),1	Yes	0,1,2	Yes
Second Forbidden	2	(1),2	No	2,3	No
Third Forbidden	3	(2),3	Yes	3,4	Yes
Fourth Forbidden	4	(3),4	No	4,5	No

- First forbidden:
  ℓ=1, Δπ= yes ⇨ S=0 (F), ⇨ S=1 (GT) } ΔI= 0, 1, 2 (unique)
- ℓ=2, Δπ= no ⇨ second forbidden etc.

All forbidden pure GT transitions (|ΔI|= ℓ+1), are called **unique**

longer $t_{1/2}$

super-allowed: log ft ≈ 3.5
allowed: log ft ≈ 4 -6
first forbidden: log ft ≈ 7- 9
second forbidden: log ft ≈ 10-13
third forbidden: log ft ≈ 14-20
fourth forbidden: log ft ≈ 23

3.5 Spontaneous fission

A spontaneous breakdown of a big nucleus into smaller nuclei and a few isolated nucleons. A nucleus is instable against fission if the Q value is higher than the fission barrier.

$$Q = T_f - T_a = (m_i - m_f)c^2$$

Fission energy can be defined as

$$\begin{aligned}\frac{E_f}{c^2} &= m(Z,A) - 2m(\frac{Z}{2}, \frac{A}{2}) \\ &= -bA^{2/3}(1-2^{1/3}) - \frac{dZ^2}{A^{\frac{1}{3}}}(1-2^{-2/3}) \\ &= \left(-5.12\, A^{2/3} + 0.284\, \frac{Z^2}{A^{\frac{1}{3}}}\right) \text{ mu}\end{aligned}$$

For symmetric fission:

$$V_C = \frac{e^2\left(\frac{Z}{2}\right)^2}{2r_0\left(\frac{A}{2}\right)^{1/3}} = C \cdot \frac{Z^2}{A^{1/3}}$$

Nuclei with $\frac{Z^2}{A} \geq 51$ are fissile
Fissility is given by:

$$x = \frac{Z^2}{51 \cdot A}$$

3.6 Excited states

3.6.1 Proton scattering

Mono energetic proton beam on a thin target (low probability of multiple scattering). Measuring proton energy distribution at a fixed angle.

$$E = E_i\left(1 - \frac{m_p}{m_A^*}\right) - E_f\left(1 + \frac{m_p}{m_A^*}\right) + \frac{2m_p}{m_A^*}\left(E_i E_f\right)^{1/2}\cos\theta$$

With $m_A^* = m_A + E/c^2$

3.6.2 Mirror nuclei

Mirror nuclei are a pair of isotopes of two different elements where the number of protons of isotope one (Z_1) equals the number of neutrons of isotope two (N_2) and the number of protons of isotope two (Z_2) equals the number of neutrons in isotope one (N_1); in short: $Z_1 = N_2$ and $Z_2 = N_1$. This implies that the mass numbers of the isotopes are the same: $N_1 + Z_1 = N_2 + Z_2$.

3.6.3 General features excited states

Many excited states correspond to several nucleon excitations.

3.6.4 Density of states

The integrated density of single neutron states gives $\mathcal{N}(E) \sim E^{3/2}$ for neutrons or protons
The number of single-nucleon states $\Delta\mathcal{N}$ in a small energy range ΔE is given by

$$\frac{\Delta\mathcal{N}}{\Delta E} \approx \frac{d\mathcal{N}}{dE} = \frac{3}{2}\frac{\mathcal{N}}{E}$$

Taking $\Delta\mathcal{N} = 1$; the mean spacing between single-particle neutron levels at the Fermi energy (38 MeV): $\mathcal{N}(E_F) = N$

$$\Delta E = \frac{2}{3}\frac{E_F}{\mathcal{N}} \sim \frac{25}{N} \text{ MeV}$$

3.7 Gamma radiation

A nucleus in an excited state de-excites by emission of high energetic photons (γ-rays)
γ-photons are emitted by the nucleus, x-rays are emitted from the atomic shell.

Photons have angular momentum - the sum of their intrinsic and orbital angular moment
Spin of the photon is one, the total angular momentum quantum number j of the photon is an integer number, $j = 0$ is not possible
Nucleus decays and changes its spin from j_i to j_f, angular momentum needs to be conserved:

$$j_i + j_f \geq j \geq |j_i - j_f|$$

γ-ray transition between states with $j_i = 0$ and $j_f = 0$ are absolutely forbidden

Parity is conserved in electromagnetic transitions; photon parity is positive if the final and initial state have the same parities, photon parity is negative if the final and initial state have opposite parities
Photon parity $(-1)^j$: "electric decay" nucleus couples with the electric field of the photon
Photon parity $-(-1)^j$: "magnetic decay" nucleus couples with the magnetic field of the photon

3.7.1 Selection rules for γ decay

Multipolarity	Dipole		Quadrupole		Octupole		.....
Type of radiation	*E1*	*M1*	*E2*	*M2*	*E3*	*M3*	
Parity change	Yes	No	No	Yes	Yes	No	

3.7.2 Partial decay rates and partial widths

Decay channels: excited states can have several options to decay into lower lying states
Each decay channel has a partial decay rate $1/\tau_i$, the total decay rate is

$$\frac{1}{\tau} = \sum_i \frac{1}{\tau_i}$$

τ is the mean life time.
Partial width of the channel is defined by $\Gamma_i = \hbar/\tau_i$. The total width is

$$\Gamma = \frac{\hbar}{\tau} = \sum_i \Gamma_i$$

Partial decay rates of nuclei for γ emission is about 10^{-15} s, thus the partial width is less than 5 eV.

4 Nuclear reactions

4.1 Cross sections

The cross section of a nuclear reaction is σ and it has the unit barn:

$$1 \text{ b} = 10^{-28} \text{ m}^2 = 10^{-24} \text{ cm}^2$$

4.2 Differential cross sections

Cross section of elastic scattering can be subdivided into sub-cross sections:
The probability $p_e(\theta,\phi)\mathrm{d}\Omega$ that a neutron is scattered into a small solid angle $\mathrm{d}\Omega = \sin\theta \mathrm{d}\theta\, \mathrm{d}\phi$ at a polar angle θ and azimuthal angle ϕ, is

$$p_e(\theta,\phi)\mathrm{d}\Omega = \frac{1}{\sigma_e}\left(\frac{\mathrm{d}\sigma_e}{\mathrm{d}\Omega}\right)\mathrm{d}\Omega$$

The elastic differential cross section is

$$\int \left(\frac{\mathrm{d}\sigma_e}{\mathrm{d}\Omega}\right)\mathrm{d}\Omega = \sigma_e$$

normally measured in Lab frame, direction of neutron in CM frame and angular dependence of the cross section will be different.

4.3 Reaction rates

Reaction rate per nucleus:

$$\text{flux} \cdot \text{cross section}$$

Experimentally per target area, reaction rate:

$$\text{flux} \cdot \text{number of target nuclei} \cdot \text{cross section}$$

4.4 The laboratory frame vs. the center of mass frame

Consider the reaction:

$$A_1 + A_2 \to A_3 + A_4$$

A_1 is the projectile (moving), A_2 is the target (stationary).

4.4.1 Laboratory frame

$$E_{\text{lab}} = \frac{m}{2} A_1 v_0^2$$

m is the nucleon mass 925 MeV, v_0 is the initial velocity.

$$p_{\text{lab}} = m A_1 v_0$$

4.4.2 Center of mass frame

Reduced mass:

$$\mu = m\frac{A_1 \cdot A_2}{A_1 + A_2} = m A_{12}$$

$$v_1 = \frac{A_2}{A_1 + A_2} \cdot v_0 \qquad\qquad v_2 = \frac{A_1}{A_1 + A_2} \cdot v_0$$

$$E_{\text{CM}} = \frac{m}{2}\left(A_1 v_1^2 + A_2 v_2^2\right) = \frac{\mu}{2} v_0^2$$
$$p_{\text{CM}} = m\left(A_1 v_1 + A_2 v_2\right) = \mu v_0$$

4.5 Rutherford scattering

If an proton passes a fixed target with a nucleus of charge Ze, it is deflected in an angle

$$\theta = \left(\frac{Ze^2}{4\pi\epsilon_0}\right)\frac{2}{bpv}$$

p is the momentum and v the velocity of the proton, b is the impact parameter.
The impact parameter being between b and $b + \mathrm{d}b$, correspond to a scattering angle between θ and $\theta - \mathrm{d}\theta$, with

$$\mathrm{d}\theta = \frac{\mathrm{d}b}{b^2}\left(\frac{2Ze^2}{4\pi\epsilon_0 pv}\right)$$

The effective area for the proton $2\pi b\,\mathrm{d}b$ contributes to the elastic scattering cross section

$$\mathrm{d}\sigma_e = 2\pi b\,\mathrm{d}b = 2\pi\left(\frac{2Ze}{4\pi\epsilon_0 pv}\right)^2\frac{\mathrm{d}\theta}{\theta^3}$$

Differential cross section for small angles:

$$\frac{\mathrm{d}\sigma_e}{\mathrm{d}\Omega} = \frac{1}{2\pi\sin\theta}\frac{\mathrm{d}\sigma_e}{\mathrm{d}\theta} = \left(\frac{2Ze}{4\pi\epsilon_0 pv}\right)^2\frac{1}{\theta^4} \qquad \text{(Rutherford scattering formula)}$$

4.6 Breit-Wigner formula

The Breit-Wigner formula gives the total cross section around the resonance energy E_0

$$\sigma_{\mathrm{tot}}(E) = \frac{\pi}{k^2}\frac{g\Gamma_i\Gamma}{(E-E_0)^2 + \Gamma^2/4}$$

$k = |\mathbf{k}|$, $\mathbf{k}$ is the wave vector of the incoming neutron in the center of mass frame, Γ_i is the partial width for decay into the incident channel $^{16}\mathrm{O} + \mathrm{n}$, g is the statistical factor (here: $g = (2j+1)/2$ where j is the spin of the excited state).
For $\Gamma \ll E_0$, the cross section is at a maximum when $E = E_0$ and the FWHM is at $E = E_0 \pm \Gamma/2$

4.7 Charged particle reactions

4.7.1 Classical approach

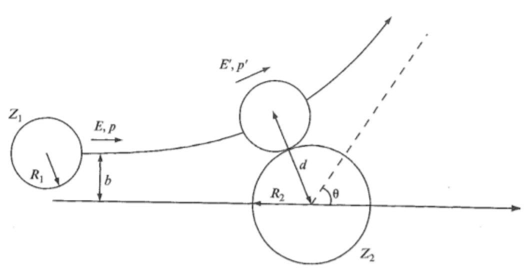


Figure 4.4 Classical grazing trajectory of a charged particle, effective radius R_1 and charge Z_1e, incident with impact parameter b at energy E and momentum p and deflected through an angle θ by a target nucleus, effective radius R_2 and charge Z_2e. The distance of closest approach d for this trajectory is the interaction radius R $(= R_1 + R_2)$ at which point the projectile has energy E' and momentum p'. The mass of the target is assumed to be large compared with the projectile mass.

$$E = E' + V_C$$

Applying conversation of angular momentum

$$L = pb = p'R$$

with b as the impact parameter.
The cross section is given by

$$\sigma = \pi R^2 \frac{E'}{E} = \pi R^2 \left(1 - \frac{V_C}{E}\right) \quad \text{for } E \geq V_C$$

Written in terms of wave number of the projectile and the angular momentum quantum number

$$\sigma = \pi \left(\frac{L}{p}\right)^2 = \frac{\pi l(l+1)\hbar^2}{(\hbar k)^2} \approx \frac{\pi l^2}{k^2} \quad \text{(assumption } l \gg 1\text{)}$$

4.7.2 Quantum mechanical consideration

Particles with a momentum p have an angular momentum between $pl\bar{\lambda}$ and $p(l+1)\bar{\lambda}$
The partial wave cross section in the l-th zone is

$$\sigma_l = (2l+1)\pi\bar{\lambda}^2$$

4.8 Reaction rates

Charged particle reactions. The total repulsive potential for the fusion of 2 charged particles is:

$$V(r) = \frac{Z_1 Z_2 e^2}{r} + \frac{l(l+1)\hbar^2}{2\mu r^2}$$

The tunneling probability at low energies and $l = 0$ is given by:

$$P = \exp\left[-2\pi\eta\right]$$

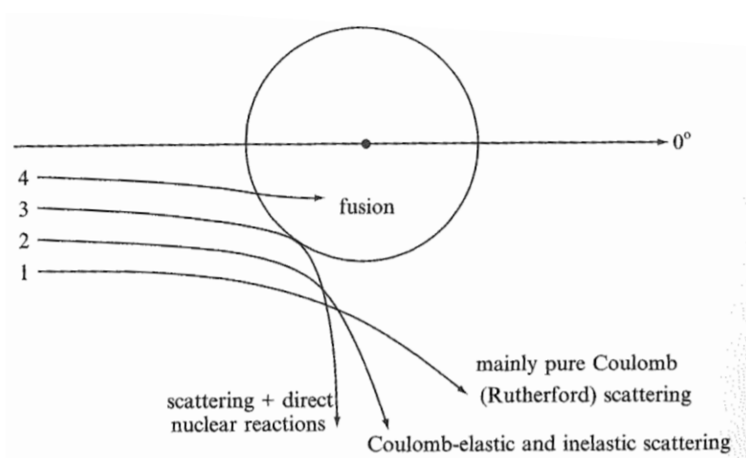
With the Sommerfeld parameter

$$\eta = \frac{Z_1 Z_2 e^2}{\hbar \nu}$$

The cross section is given by:

$$\sigma(E) = \frac{1}{E} S(E) \exp\left[-2\pi\eta\right]$$

4.9 Heavy ion reactions

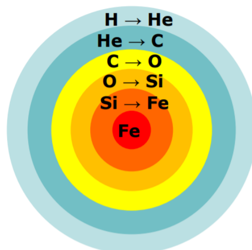


4.9.1 Formation of heavy elements

$$\sigma_{EVR}(E) = \frac{\pi h^2}{2\mu E} \sum_{l=0}^{\infty} (2l+1)\, T(E,l)\, P_{CN}(E,l)\, W_{sur}(E,l)$$

E: energy in CM
T: probability of colliding nuclei to overcome the Coulomb barrier
P_{CN}: probability of compound nucleus formation
W_{sur}: probability that the compound nucleus evaporates nucleons and does not fission

4.9.2 In stars



Stage	Duration
H → He	$7*10^6$ years
He → C	$7*10^5$ years
C → O	600 years
O → Si	6 months
Si → Fe	1 day
Core collapse	¼ second

4.9.3 Solar system abundances above Fe

Most noticeable features of abundance plot:

- s- and r-process abundances about the same
- p-process much lower
- r-process peaks shifted towards lower masses

The slow neutron-capture process, or s-process, is a series of reactions in nuclear astrophysics that occur in stars, particularly AGB stars. In the s-process, a seed nucleus undergoes neutron capture to form an isotope with one higher atomic mass. If the new isotope is stable, a series of increases in mass can occur.
Proton-rich nuclides can be produced by sequentially adding one or more protons to an atomic nucleus. Such a nuclear reaction of type (p,γ) is called proton capture reaction. This is the p-process.
rp-process: sequences of proton captures and beta +-decays along proton dripline.

5 Interaction of radiation with matter

Ionization:

$$M \rightarrow M^+ + e^-$$

Ion- electron pair formation

Excitations:

$$M \rightarrow M^*$$

molecular degrees of freedom (rotation, vibration, translation)

Wanted effects: detectors, shielding, particle therapy, material modification

Un-wanted effects: tissue damage, health issues, radiation damage (materials)

5.1 Bethe-Bloch formula

$$-\frac{dE}{dx} = \left(\frac{ze^2}{4\pi\epsilon_0}\right)^2 \frac{4\pi Z\rho N_A}{Am_e v^2}\left[\ln\left(\frac{2m_e v^2}{I}\right) - \ln\left(1-\beta^2\right) - \beta^2\right]$$

Energy transfer $\left(\frac{p^2}{m_e}\right)$ to an electron depends on the electric force $\propto \left(\frac{ze^2}{4\pi\epsilon_0}\right)^2$, the square of the transit time $\propto \left(\frac{1}{v^2}\right)$ and $\left(\frac{1}{m_e}\right)$

Collision rate is proportional to the density of electrons in the material: $\frac{Z\rho N_A}{A}$

Last part comes from the impact parameter -amount of energy transferred